

# $S_4$ as a natural flavor symmetry for lepton mixing

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Group theoretical arguments seem to indicate the discrete symmetry  $S_4$  as the minimal flavour symmetry compatible with tribimaximal neutrino mixing. We prove in a model independent way that indeed  $S_4$  can realize exact tribimaximal mixing through different symmetry breaking patterns. We present two models in which lepton tribimaximal mixing is realized in different ways and for each one we discuss the superpotential that leads to the correct breaking of the flavor symmetry.

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## I. INTRODUCTION

Harrison, Perkins and Scott (HPS) [1] proposed the so called tribimaximal mixing matrix

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}. \quad (1)$$

This matrix keeps in surprising agreement with experimental data [2]. Lot of theoretical models has been done to explain the mixing matrix of eq. (1) by means of non abelian flavor symmetry, such as  $S_3$  [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13],  $A_4$  [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29],  $T'$  [30, 31, 32, 33, 34],  $S_4$  [35, 36, 37, 38, 39] and  $\Delta(27)$  [40, 41, 42, 43]. The non abelian discrete groups have irreducible representations of dimension bigger than one [44]. The most interesting case arises when the group contains a triplet as irreducible representation, allowing to embed the observed three generations of fermions.

When a non abelian discrete group  $G$  is broken to one of its subgroup  $G'$  the transformation  $U_{G'}$  that decomposes the representations of  $G$  according to  $G'$  can be fixed and are completely model independent. This is the case for example of  $A_4$  broken to  $Z_3$ : the triplet representation of  $A_4$  is sent to the one-dimensional representations of  $Z_3$ ,  $1, 1', 1''$ , through the matrix  $U_\omega$  defined as

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (2)$$

while the one dimensional representations of  $A_4$  coincide with the corresponding ones of  $Z_3$ . A good candidate to give TBM is a discrete group  $G$  that has a triplet representation, at least two subgroups,  $G'$  that decompose according to  $U_{G'}$  and  $G''$  that decompose according to  $U_{G''}$ . It is necessary having at least two different subgroups of  $G$  to obtain a lepton mixing matrix different to the identity: if  $G$  were broken to the same subgroup  $G'$  both in the charged lepton and in the neutrino sector the lepton mixing matrix would be given by  $U_{lep} = U_{G'}^\dagger, U_{G'} = \mathcal{I}$ .

*A priori*  $A_4$  seems to be a good candidate because it is the smallest discrete group that contains a triplet as irreducible representation. Furthermore it has two different subgroups,  $Z_3$  and  $Z_2$ . However, while the transformation associated to  $Z_3$  is given by  $U_\omega$  the one associated to  $Z_2$  is model dependent. This analysis has been already performed in [45] (see eq. A4). A similar analysis done with the discrete symmetry  $T'$  lead to the same conclusion (see eq. (8) of Ref.[30]). This means that  $A_4$  and  $T'$  yield exact or approximate TBM only assuming a fine tuning in the parameters of the Yukawa lagrangian or a particular model realization. We mention that by assuming further constraints, also models based on  $S_3$  can yield an approximate TBM, although its largest irreducible representation is a doublet and not a triplet.

It has been recently claimed [46] that the minimal flavor symmetry naturally related to the tribimaximal mixing is  $S_4$ , the permutation symmetry of four objects. The author of [46] proved this through group theoretical arguments without entering into the details of a concrete model realization. In this paper we provide a concrete model realization

of these general arguments reconsidered  $S_4$  and its subgroups. We have found that  $S_4$  is able to reproduce TBM following two different symmetry breaking patterns. We have built two different models that realize TBM through the two patterns dictated by the group analysis considerations and finally we discuss the possible superpotential that can break  $S_4$  in the correct way.

## II. THE DISCRETE SYMMETRY GROUP $S_4$ AS THE ORIGIN OF TBM

### A. The group $S_4$

The discrete group  $S_4$  is given by the permutations of four objects and it is composed by 24 elements. It can be defined by two generators  $S$  and  $T$  that satisfy

$$S^4 = T^3 = 1, \quad ST^2S = T. \quad (3)$$

The 24 elements of  $S_4$  belong to five classes

$$\begin{aligned} \mathcal{C}_1 &: I; \\ \mathcal{C}_2 &: S^2, TS^2T^2, S^2TS^2T^2; \\ \mathcal{C}_3 &: T, T^2, S^2T, S^2T^2, STST^2, STS, S^2TS^2, S^3TS; \\ \mathcal{C}_4 &: ST^2, T^2S, TST, TSTS^2, STS^2, S^2TS; \\ \mathcal{C}_5 &: S, TST^2, ST, TS, S^3, S^3T^2. \end{aligned} \quad (4)$$

The elements of  $\mathcal{C}_{2,4}$  define two different sets of  $Z_2$  subgroups of  $S_4$ , that ones of the class  $\mathcal{C}_4$  a set of  $Z_3$  abelian discrete symmetries and those belonging to  $\mathcal{C}_5$  a set of  $Z_4$  abelian discrete symmetries. The  $S_4$  irreducible representations are two singlets,  $1_1, 1_2$ , one doublet,  $2$ , and two triplets,  $3_1$  and  $3_2$ . We adopt the following basis

$$S = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad T = -\frac{1}{2} \begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}, \quad (5)$$

for the doublet representation and

$$S_{+,-} = \pm \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad (6)$$

for the triplet representations. Clearly the generators  $(S_+, T)$  and  $(S_-, T)$  define the two triplet representations  $3_1, 3_2$  respectively. All the product rules can be straightforwardly derived. We remind the reader to the product rules reported in [36].

### B. $S_4$ symmetry breaking patterns

We have seen in the introduction that given a discrete non abelian group  $G$  a predictive lepton mixing matrix may be obtained if  $G$  is broken to one of its subgroups, with the subgroup preserved in the charged lepton sector different from the subgroup preserved in the neutrino sector.

We disregard therefore the case when  $S_4$  is completely broken in one of the two sectors. At the same time, if the left handed leptons transform non-trivially under  $S_4$ , the case of  $S_4$  unbroken in one sector is ruled out since it leads to a diagonal mass matrix with at least two degenerate states. Therefore if  $S_4$  is broken to one of its subgroups  $G'$  in the charged lepton sector, in the neutrino sector it has to be broken to another subgroup  $G'' \neq G'$ . The couple  $(G', G'')$  identifies a possible symmetry breaking pattern. In this notation the lepton mixing matrix is given by

$$U_{lep} = U_l^\dagger U_\nu = U_{G'}^\dagger U_{G''}, \quad (7)$$

being  $U_{G'}, U_{G''}$  the transformations that decompose the representations of  $S_4$  into the representations of  $G', G''$  respectively.

$S_4$  contains a non abelian subgroup  $S_3$ , the permutation group of three objects composed by six elements. The elements of  $S_4$  that belong to  $S_3$  correspond to  $C_1$ ,  $T$  and  $T^2$  of  $C_3$  and  $TSTS^2, STS^2, S^2TS$  of  $C_4$ . Furthermore  $S_4$  contains the abelian subgroups  $Z_2$ ,  $Z_3$ ,  $Z_4$  corresponding to the elements of the classes  $\mathcal{C}_{2,4}, \mathcal{C}_3$  and  $\mathcal{C}_5$  respectively. The only representation that can break  $S_4$  to  $S_3$  is the triplet  $3_1$ . The reason is that when a triplet  $\phi_1 \sim 3_1$  develops vev as  $(1,1,1)$  the six elements that define  $S_3$  belonging to  $S_4$ — $I, T, T^2, TSTS^2, STS^2, S^2TS$  built with the basis reported in eq. (6)—are preserved. On the contrary, when a triplet  $\phi_2 \sim 3_2$  develops vev as  $(1,1,1)$ , only the three elements that define  $Z_3$  are preserved— $I, T, T^2$ —while  $TSTS^2, STS^2, S^2TS$  built according eq. (6) are broken.

The representations of  $S_3$  are two singlet,  $1_1$  and  $1_2$ , and a doublet,  $2$ . In general if  $S_4$  is broken to  $S_3$  the representations of  $S_4$  would transform under  $S_3$  according to

$$3_1 \rightarrow 1_1 + 2, \quad 3_2 \rightarrow 1_2 + 2, \quad 2 \rightarrow 2, \quad 1_1 \rightarrow 1_1, \quad 1_2 \rightarrow 1_2. \quad (8)$$

Therefore if  $S_4$  is broken to  $S_3$ , a triplet of  $S_4$ ,  $F \sim (F_1, F_2, F_3) \sim 3_1$ , will decompose under  $S_3$  as  $F(3_1) \rightarrow \psi_0(1_+) + \psi(2_-)$  with

$$\psi_0 = \frac{1}{\sqrt{3}}(F_1 + F_2 + F_3), \quad \psi = \begin{pmatrix} (F_2 - F_3)/\sqrt{2} \\ (-2F_1 + F_2 + F_3)/\sqrt{6} \end{pmatrix}. \quad (9)$$

The new eigenstates  $S_3$  ( $\psi_0, \psi$ ) are defined by

$$\begin{pmatrix} \psi_0 \\ \psi_1 \\ \psi_2 \end{pmatrix} = U_{S_3} \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \quad \text{with} \quad U_{S_3} = P \cdot U_{TBM}^T \quad \text{with} \quad P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (10)$$

We now assume that  $F \sim L$  being  $L$  the left handed lepton doublets and for the moment we leave undetermined the transformation properties under  $S_4$  of the electroweak  $SU(2)$  singlets.

The first case we consider is the symmetry breaking pattern  $(S_3, G'')$ , that means that we break  $S_4$  into  $S_3$  in the charged lepton sector while we still not know which is its corresponding  $S_4$  subgroup in the neutrino sector. Assuming that the LR charged lepton mass matrix  $M_l$  is obtained once  $S_4$  is broken to  $S_3$ , we can write  $M_l M_l^\dagger$  in the new basis defined by eq. (10)

$$M_l M_l^\dagger \rightarrow P U_{TBM}^T M_l M_l^\dagger U_{TBM} P = \tilde{M}_l \tilde{M}_l^\dagger. \quad (11)$$

Since the residual symmetry is  $S_3$ ,  $\tilde{M}_l \tilde{M}_l^\dagger$  has to be invariant under this symmetry. Once we impose this condition we discover that  $\tilde{M}_l \tilde{M}_l^\dagger = M_{diag}^l M_{diag}^{l\dagger}$ , with 2 degenerate masses. Neglecting for the moment this phenomenological inconsistency, we have seen that the breaking  $S_4 \rightarrow S_3$  in the charged lepton sector has lead to  $U_l = U_{TBM} P$ . If the neutrino mass matrix were diagonal  $U_{lep} = U_l^\dagger U_\nu$  would lead to the wrong conclusion  $U_{lep} = U_{TBM}^T$ . To cure this problem we have two options. On one hand, we could require that the neutrino mass matrix were diagonalized by  $U_{TBM} U_{TBM}$  in order to reproduce the TBM through  $U_{lep} = U_{TBM}^T U_{TBM} U_{TBM} = U_{TBM}$ . However there is no  $G''$  subgroup of  $S_4$  that yields  $U_{G''} = U_{TBM} U_{TBM}$  and therefore exact TBM cannot be obtained according to eq. (7). On the other hand we could require to break the surviving  $S_3$  in the charged lepton sector into  $Z_2$  in such a way to produce a  $U_l \neq U_{TBM} P$ . Even in this case there is no corresponding  $G''$  in the neutrino sector that allows to obtain exact TBM. As consequence the symmetry breaking pattern with  $S_4$  broken into  $S_3$  in the charged lepton sector is ruled out.

We now analyze what happens considering the breaking pattern  $(Z_3, G'')$ . As in the previous case the subgroup  $G''$ , corresponding to the neutrino sector, is undetermined. We expect that if we break  $S_4$  into  $Z_3$  in the charged lepton sector—we have already said that in  $S_4$  the breaking into  $Z_3$  is realized when a triplet  $3_2$  develops a vev in the direction  $(1,1,1)$ —the charged lepton mixing matrix will send the  $S_4$  triplet  $(L_1, L_2, L_3)$  in the  $Z_3$  eigenstates,  $1, 1', 1''$ . Indeed the mixing matrix responsible of this rotation is the  $U_\omega$  defined in eq. (2). Given  $U_\omega$  the correct TBM can be reproduced if the  $U_{G''}$  of eq. (7) is given by

$$U_\nu = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \end{pmatrix}, \quad (12)$$

or in other words if the neutrino mass matrix  $m^\nu$  is diagonalized by  $U_\nu$  and it has the following form

$$m^\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & c & b \\ 0 & b & c \end{pmatrix}. \quad (13)$$

The matrix form of eq. (13) is recovered by requiring the invariance of  $m^\nu$  under the  $G'' = Z_2$  subgroup of  $S_4$  associated to the element  $TST$  of the class  $\mathcal{C}_4$ . This breaking pattern is the usual one used in models based on  $A_4$ . However we stress that in the context of  $S_4$  we have obtained TBM only according to group theory considerations.

If we consider now the case  $(Z_2, G'')$  we discover that  $S_4$  behaves exactly as  $A_4$  and exact TBM cannot be recovered. For a detailed analysis we remand the reader to the Appendix of [45].

In the case  $(Z_4, G'')$  we discover that the charged lepton mass matrix  $M_l M_l^\dagger$  is diagonal with two states that are degenerate. Since  $Z_4$  is abelian this degeneration can be broken only by completely breaking  $Z_4$ . In this case  $U_{G'}$  of eq. (7) completely arbitrary and exact TBM cannot be obtained.

So far we have considered all the possible cases in which the subgroup fixed in the charged lepton sector gives rise to a non diagonal structure to the charged lepton mass matrix  $M_l$ . The last case involving  $Z_4$  gives rise to a diagonal  $M_l M_l^\dagger$  but with two degenerate states. We could ask if there is any way to realize a diagonal  $M_l$  with three different mass eigenvalues. Indeed this is easily realized breaking  $S_4$  to  $Z_2 \times Z_2$  corresponding to the elements  $S^2$  and  $T^2 S^2 T$  of the class  $\mathcal{C}_2$ . If the charged lepton mass matrix is diagonal all the mixing structure arise by the neutrino sector. Therefore the last symmetry breaking pattern we are going to consider is  $(Z_2 \times Z_2, S_3)$ .

In this last case we break  $S_4$  into  $S_3$  in the neutrino sector. Following the same analysis that brought to eq. (10) we have

$$U_{TBM}^T m^\nu U_{TBM} = m_{S_3}^\nu, \quad (14)$$

that means  $U_{lep} = U_{TBM}$  being the charged lepton mass matrix diagonal. At this point we have to face off a further problem: when  $S_4$  is broken to  $S_3$  the triplet  $L$  splits in a singlet plus a doublet. If  $S_3$  is unbroken the two states in the doublet are degenerate in contrast with experimental data. Therefore we should identify a way of breaking  $S_3$  without affecting the mixing rotation of the neutrino mass matrix. To keep us as general as possible, consider  $m_{S_3}^\nu$  obtained once  $S_4 \rightarrow S_3$ . If  $S_3$  is unbroken we have  $m_{S_3}^\nu = \text{Diag}(m_1, m_0, m_0)$ .

Suppose now that the singlet and the doublet with respect to  $S_3$  behave as two independent sectors in such a way that  $S_3$  is preserved in the singlet sector while is broken in the doublet one<sup>1</sup>. By imposing these conditions we discover that  $m_{S_3 \text{ broken}}^\nu$  has the following expression

$$m_{S_3 \text{ broken}}^\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & b_1 & b_2 \\ 0 & b_2 & b_3 \end{pmatrix}. \quad (15)$$

Finally let us impose that  $S_3$  is not completely broken in the doublet sector but it is broken to its subgroup  $Z_2$  identified by the  $S_3$  generator  $S$ . This generator coincides with the  $S$  generator of the doublet representation of  $S_4$  given in eq. (5). In this case it is possible to show that  $m_{S_3 \text{ broken}}^\nu = \text{Diag}(m_1, m_2, m_3)$  and the lepton mixing matrix is still given by  $U_{TBM}$ .

We have seen that on the basis of theoretical considerations based on the subgroups of  $S_4$ , the flavor symmetry  $S_4$  has two symmetry breaking patterns giving exact TBM in the lepton sector. In the next section we will present a model realization for each breaking pattern. In the last section we build the corresponding supepotential responsible for the correct  $S_4$  symmetry breaking patterns.

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<sup>1</sup> From the point of view of model realization this assumption is not different by assuming that  $S_4$  is broken to different subgroups in the charged lepton sector and in the neutrino one. Indeed we will see in sec. III B how singlet and doublet sectors can be easily separated.

### III. MODEL REALIZATION

#### A. Model I : $S_4 \rightarrow Z_3$ & $S_4 \rightarrow Z_2$

The first model we consider reproduces TBM through the breaking of  $S_4$  into  $Z_3$  and  $Z_2$  in the charged lepton and neutrino sector respectively. We assume our model to be supersymmetric. Matter and scalar supermultiplets are reported in tab. I. The scalar supermultiplets charged under  $S_4$ , that in the following we will identify as flavons, are electroweak  $SU(2) \times U(1)$  singlets. Therefore the Yukawa superpotential  $\mathcal{W}_Y$  of eq. (16) includes effective operators of order 4.  $\Lambda$  is the cutoff of the model and an extra  $Z_5$  symmetry has been introduced to separate the charged lepton sector from the neutrino one. In tab. I we have omitted the supermultiplets  $\hat{H}^u$  and  $\hat{\Phi}$ , doublet and triplet of  $SU(2)$  respectively, necessary to give mass to the up-quarks and to cancel anomalies in a realistic model.

	$\hat{L}$	$\hat{E}^c$	$\hat{H}^d$	$\hat{\Phi}$	$\hat{\sigma}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\Delta}$
$SU(2)$	2	1	2	3	1	1	1	1
$S_4$	3 <sub>1</sub>	3 <sub>1</sub>	1	1	1	3 <sub>1</sub>	3 <sub>2</sub>	3 <sub>1</sub>
$Z_5$	1	$\omega_5^4$	1	1	$\omega_5$	$\omega_5$	$\omega_5$	1

TABLE I: Matter and scalar content of model I. The lepton mixing matrix is TB.

The full leading order  $S_4 \times Z_5$  Yukawa superpotential  $\mathcal{W}_Y$  is given by

$$\mathcal{W}_Y = \frac{1}{\Lambda} y_0 (\hat{L} \hat{E}^c)_1 \hat{\sigma} \hat{H}^d + \frac{1}{\Lambda} y_s (\hat{L} \hat{E}^c)_{3_1} \hat{\phi}_1 \hat{H}^d + \frac{1}{\Lambda} y_a (\hat{L} \hat{E}^c)_{3_2} \hat{\phi}_2 \hat{H}^d + y'_1 (\hat{L} \hat{L})_1 \hat{\Phi} + \frac{1}{\Lambda} y'_2 (\hat{L} \hat{L})_{3_1} \hat{\Delta} \hat{\Phi}. \quad (16)$$

When the  $S_4$  triplet and doublet flavons align as

$$\langle \phi_1 \rangle \sim \langle \phi_2 \rangle \sim (1, 1, 1) \quad \langle \Delta \rangle \sim (1, 0, 0), \quad (17)$$

the charged lepton and neutrino mass matrices present the usual forms

$$M_l = \begin{pmatrix} h_0 & h_1 & h_2 \\ h_2 & h_0 & h_1 \\ h_1 & h_2 & h_0 \end{pmatrix} \quad m^\nu = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix} \quad (18)$$

that satisfy

$$U_\omega M_l U_\omega^\dagger = M_l^{diag}, \quad U_\nu^T m_\nu U_\nu = m_\nu^{diag}, \quad (19)$$

with  $U_\omega$  and  $U_\nu$  given in eq. (2) and eq. (12) respectively. TBM is obtained as usual by  $U_{TB} = U_\omega U_\nu$ . The mass eigenvalues for the charged lepton are given by

$$m_e = h_0 + h_1 + h_2, \quad m_\mu = h_0 + h_1 \omega^2 + h_2 \omega, \quad m_\tau = h_0 + h_1 \omega + h_2 \omega^2, \quad (20)$$

and for the neutrino by  $(a + b, a, b - a)$ . By assuming that the flavon vevs are of order  $\sim \lambda^2 \Lambda$  with  $\lambda$  the Cabibbo angle, the deviations from TBM induced by the next to leading order corrections to the Yukawa superpotential slightly modify lepton mixing keeping it still in agreement with neutrino data. Notice that the vev alignments

$$\langle \phi_1 \rangle \sim \langle \phi_2 \rangle \sim (1, 1, 1) \quad (21)$$

preserves the  $Z_3$  subgroup of  $S_4$  associated to the element  $T$ , while the vev alignments

$$\langle \varphi \rangle \sim (0, 1) \quad \langle \Delta \rangle \sim (1, 0, 0), \quad (22)$$

preserves the  $Z_2$  associated to the element  $TST$  that in the doublet and triplet representation reads respectively as

$$TST = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad TST = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \quad (23)$$

### B. Model II : $S_4 \rightarrow S_3$

The second model we describe realizes TBM through the sequential breaking of  $S_4$  into  $S_3$  and then into  $Z_2$  in the neutrino sector and the breaking of  $S_4$  into two different  $Z_2 \times Z_2$  in the charged lepton sector. The step through  $S_3$  is crucial : if we broke  $S_4$  directly into  $Z_2$  in the neutrino sector we would find a generic neutrino mass matrix  $\mu - \tau$  invariant not diagonalized by TB. On the contrary, in the model that we present the step through  $S_3$  leads to a neutrino mass matrix  $m^\nu$  which is  $\mu - \tau$  invariant and satisfy the relation  $m_{11}^\nu = m_{22}^\nu + m_{23}^\nu - m_{13}^\nu$  that ensures TB diagonalization. We will see that the key ingredient in building the correct  $m^\nu$  is the introduction of the right handed neutrinos transforming as a doublet of  $S_4$ . As in the case of the model presented in sec. III A we assume our model be supersymmetric and the flavon supermultiplets electroweak singlets. Matter and scalar supermultiplets are reported in tab. II. As done in sec. III A we have omitted the supermultiplet  $\hat{\Phi}$ , triplet of  $SU(2)$ , necessary to cancel anomalies. Two extra discrete abelian symmetries,  $Z_3$  and  $Z_5$ , have been introduced in order to avoid interferences between the sectors.

	$\hat{L}$	$\hat{l}^c$	$\hat{N}^c$	$\hat{H}^u$	$\hat{H}^d$	$\hat{\Phi}$	$\hat{\Delta}$	$\hat{\sigma}$	$\hat{\phi}$	$\hat{\varphi}$
$SU(2)$	2	1	1	2	2	3	1	1	1	1
$S_4$	$3_1$	$3_1$	2	1	1	1	$3_1$	1	2	2
$Z_3$	$\omega^2$	1	1	1	$\omega^2$	$\omega$	$\omega$	1	1	1
$Z_5$	1	$\omega_5^3$	1	1	1	1	1	$\omega_5^2$	$\omega_5^2$	1

TABLE II: Matter and scalar content of model II. The lepton mixing matrix is TB.

The full leading order  $S_4 \times Z_3 \times Z_5$  invariant Yukawa superpotential is given by

$$\mathcal{W}_Y = \frac{1}{\Lambda} y_s (\hat{L} \hat{l}^c)_1 \hat{\sigma} \hat{H}^d + \frac{1}{\Lambda} y_d (\hat{L} \hat{l}^c)_2 \hat{\phi} \hat{H}^d + y_1 (\hat{L} \hat{L})_1 \hat{\Phi} + \frac{1}{\Lambda} y_2 (\hat{L} \hat{\Delta})_2 \hat{N}^c \hat{H}^u + M_d \hat{N}^c \hat{N}^c + \tilde{y}_N \hat{\varphi} \hat{N}^c \hat{N}^c, \quad (24)$$

where as usual  $\Lambda$  is the cutoff of the model and all the Yukawa terms are of order 4 with the exception of the ones involving right handed neutrinos. We assume that the flavons  $\Delta$  and  $\varphi$ , triplet and doublet under  $S_4$  respectively, align as

$$\langle \Delta \rangle \sim (1, 1, 1) \quad \langle \varphi \rangle \sim (0, 1). \quad (25)$$

The vev  $\langle \Delta \rangle$  preserves  $S_3$  as has been already discussed in sec. IIB. The vev  $\langle \varphi \rangle$  preserves the  $S$  generators of  $S_3$  that coincides with the  $S$  generator of  $S_4$  of the doublet representation—eq. (5).

The doublet  $\phi$  does not align and develops vev as  $\langle \phi \rangle \sim (v_1, v_2)$ —this means that  $S_4$  is broken to  $Z_2 \times Z_2$  corresponding to the elements  $S^2$  and  $T S^2 T^2$  of  $\mathcal{C}_2$  that in the  $3_1$  triplet representation read as

$$S^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T S^2 T^2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (26)$$

For the charged lepton sector we have

$$M_l = \frac{1}{\Lambda} v^d \begin{pmatrix} y'_s v_\sigma - 2y''_d v_2^\phi & 0 & 0 \\ 0 & y'_s v_\sigma + y'_d v_1^\phi + y''_d v_2^\phi & 0 \\ 0 & 0 & y'_s v_\sigma - y'_d v_1^\phi + y''_d v_2^\phi \end{pmatrix} \quad (27)$$

with  $v_\sigma = \langle \sigma \rangle$   $v_{1,2}^\phi = \langle \phi_{1,2} \rangle$   $v^d = \langle H_0^d \rangle$ , and the product factors absorbed in  $y'_s$  and  $y'_d, y''_d$ . The neutrino mass matrix gets contributions both from type I and type II see-saw

$$m^\nu = m_{LL} - m_D \cdot \frac{1}{M_N} \cdot m_D^T, \quad (28)$$

where  $m_{LL} = y_1 v_\Phi \cdot \mathcal{I}$  with  $v_\Phi = \langle \Phi \rangle$  and

$$m_D = y_2 \frac{v^\Delta}{\Lambda} v^u \begin{pmatrix} 0 & -2\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{6} \\ -1/\sqrt{2} & 1/\sqrt{6} \end{pmatrix}, \quad M_N = \begin{pmatrix} M_d + V_\varphi & 0 \\ 0 & M_d - V_\varphi \end{pmatrix}, \quad (29)$$

with  $v^u = \langle H_0^u \rangle$ ,  $v_{1,2,3}^\Delta = v^\Delta$  and  $V_\varphi = \tilde{y}_N \langle \varphi_2 \rangle / \sqrt{2}$ . After the usual see-saw mechanism the majorana neutrino mass matrix is given by

$$m^\nu = \begin{pmatrix} a + \frac{2}{3}b & -\frac{1}{3}b & -\frac{1}{3}b \\ -\frac{1}{3}b & a + \frac{1}{6}b + \frac{1}{2}c & \frac{1}{6}b - \frac{1}{2}c \\ -\frac{1}{6}b & \frac{1}{6}b - \frac{1}{2}c & a + \frac{1}{6}b + \frac{1}{2}c \end{pmatrix}, \quad (30)$$

with

$$a = y_1 v_\Phi, \quad b = -y_2^2 \left( \frac{v^\Delta}{\Lambda} \right)^2 \frac{(v^u)^2}{M_d - V_\varphi}, \quad c = -y_2^2 \left( \frac{v^\Delta}{\Lambda} \right)^2 \frac{(v^u)^2}{M_d + V_\varphi}. \quad (31)$$

The neutrino mass matrix  $m^\nu$  is diagonalized by TBM and its eigenvalues are  $(a+b, a, a+c)$  that can accommodate experimental neutrino mass splitting data being expressed in terms of three independent combinations of the parameters of the model. As in the model discussed in sec. III A by assuming the flavon vevs of order  $\sim \lambda^2 \Lambda$  next to leading order corrections to the Yukawa superpotential produce small deviations from TBM that are still compatible with neutrino data.

#### IV. REALIZING THE CORRECT VACUUM CONFIGURATIONS IN $S_4$

In the context of flavor model based on non abelian discrete symmetry the lepton TBM is obtained thanks to specific alignments of the flavons. The so-called alignment problem in  $A_4$  and  $T'$  has been extensively discussed in [18, 21, 25]. Different strategies have been used: the introduction of soft breaking term of the flavor symmetry [25], the use of a continuous  $U(1)_R$  symmetry [21] preserved by the scalar potential and the promotion of the model to a fifth dimension [18]. In the context of  $S_4$  in [39] the flavon superpotential was softly broken to guarantee the desired vacuum configuration.

In  $S_4$  as well as in  $A_4$  and  $T'$  it is impossible to build a flavon superpotential that guarantees the alignments needed. In the next sections we will show that the extra discrete abelian symmetries introduced in sec. III to separate the two lepton sectors are sufficient to give the correct vacuum configurations.

##### A. Model I : minimization of the potential

	$\hat{\sigma}$	$\hat{\phi}_1$	$\hat{\phi}_2$	$\hat{\Delta}$	$\hat{\varphi}$	$\hat{\xi}$	$\hat{\eta}$
$SU(2)$	1	1	1	1	1	1	1
$S_4$	1	3_1	3_2	3_1	2	2	2
$Z_5$	$\omega_5$	$\omega_5$	$\omega_5$	1	1	$\omega_5^3$	$\omega_5^2$

TABLE III: Scalar content of model I including the flavons that contribute to the mass matrix structures and the ones the drive the correct vacuum alignments, the driving fields.

The flavon potential is obtained by the following part of the full  $S_4 \times Z_5$  superpotential

$$\begin{aligned} \mathcal{W}_Y = & M_{\xi\eta} \hat{\xi} \hat{\eta} + \lambda_{\xi\eta} \hat{\xi} \hat{\eta} \hat{\varphi} + \lambda_{\sigma\eta} \hat{\sigma} \hat{\eta} \hat{\eta} + \lambda_{\xi\phi_1} \hat{\xi} \hat{\phi}_1 \hat{\phi}_1 + \lambda_{\xi\phi_2} \hat{\xi} \hat{\phi}_2 \hat{\phi}_2 + \lambda_{\xi\phi_1 2} \hat{\xi} \hat{\phi}_1 \hat{\phi}_2 \\ & + M_\Delta \hat{\Delta} \hat{\Delta} + M_\varphi \hat{\varphi} \hat{\varphi} + \lambda_{\varphi\Delta} \hat{\Delta} \hat{\Delta} \hat{\varphi} + \lambda_\varphi \hat{\varphi} \hat{\varphi} \hat{\varphi} + \lambda_\Delta \hat{\Delta} \hat{\Delta} \hat{\Delta}. \end{aligned} \quad (32)$$

We assume that the flavor symmetry is broken in the SUSY limit and therefore the vacuum configuration is obtained solving the system  $\partial \mathcal{W}_Y / \partial f_i = 0$ , where  $f_i$  are the  $f$  components of the supermultiplets entering in eq. (32) and  $i$  runs on all the supermultiplets. By assuming the general vacuum configuration

$$\langle \Delta \rangle = (v_1^\Delta, v_2^\Delta, v_3^\Delta), \quad \langle \varphi \rangle = (v_1^\varphi, v_2^\varphi), \quad \langle \phi_1 \rangle = (v_1^\phi, v_2^\phi, v_3^\phi), \quad \langle \phi_2 \rangle = (u_1^\phi, u_2^\phi, u_3^\phi), \quad \langle \xi \rangle = (u_1^\xi, u_2^\xi), \quad \langle \eta \rangle = (z^\eta, z^\eta) \quad \langle \sigma \rangle = v_\sigma, \quad (33)$$

the set of equations is given by

$$\begin{aligned}
a) \quad \partial W / \partial f_1^\Delta &= \frac{2}{\sqrt{3}} M_\Delta v_1^\Delta - \frac{2}{\sqrt{3}} \lambda_{\Delta\varphi} v_1^\Delta v_2^\varphi + 2\lambda_\Delta v_2^\Delta v_3^\Delta = 0 \\
b) \quad \partial W / \partial f_2^\Delta &= \frac{2}{\sqrt{3}} M_\Delta v_2^\Delta + \frac{1}{\sqrt{3}} \lambda_{\Delta\varphi} v_2^\Delta (v_2^\varphi + \sqrt{3}v_1^\varphi) + 2\lambda_\Delta v_1^\Delta v_3^\Delta = 0 \\
c) \quad \partial W / \partial f_3^\Delta &= \frac{2}{\sqrt{3}} M_\Delta v_3^\Delta + \frac{1}{\sqrt{3}} \lambda_{\Delta\varphi} v_3^\Delta (v_2^\varphi - \sqrt{3}v_1^\varphi) + 2\lambda_\Delta v_1^\Delta v_2^\Delta = 0 \\
d) \quad \partial W / \partial f_1^\varphi &= \sqrt{2} M_\varphi v_1^\varphi + \frac{\lambda_{\xi\eta}}{2} (u_2^\xi z_1^\eta + u_1^\xi z_2^\eta) + \frac{\lambda_\Delta}{2} [(v_2^\Delta)^2 - (v_3^\Delta)^2] = 0 \\
e) \quad \partial W / \partial f_2^\varphi &= \sqrt{2} M_\varphi v_2^\varphi + \frac{\lambda_{\xi\eta}}{2} (u_1^\xi z_1^\eta - u_2^\xi z_2^\eta) + \frac{\lambda_\Delta}{2\sqrt{3}} [-2(v_1^\Delta)^2 + (v_2^\Delta)^2 + (v_3^\Delta)^2] = 0 \\
f) \quad \partial W / \partial f_1^\eta &= \frac{M_{\xi\eta}}{\sqrt{2}} u_1^\xi + \frac{\lambda_{\xi\eta}}{2} (v_1^\varphi u_2^\xi + v_2^\varphi u_1^\xi) + \sqrt{2} \lambda_{\sigma\eta} v_\sigma z_1^\eta = 0 \\
g) \quad \partial W / \partial f_2^\eta &= \frac{M_{\xi\eta}}{\sqrt{2}} u_2^\xi + \frac{\lambda_{\xi\eta}}{2} (v_1^\varphi u_1^\xi - v_2^\varphi u_2^\xi) + \sqrt{2} \lambda_{\sigma\eta} v_\sigma z_2^\eta = 0 \\
h) \quad \partial W / \partial f^\sigma &= \frac{\lambda_{\sigma\eta}}{\sqrt{2}} [(z_1^\eta)^2 + (z_2^\eta)^2] = 0 \\
i) \quad \partial W / \partial f_1^\xi &= \frac{1}{\sqrt{2}} M_{\xi\eta} z_1^\eta + \frac{1}{2} \lambda_{\xi\eta} (z_1^\eta v_2^\varphi + z_2^\eta v_1^\varphi) + \frac{1}{2} \lambda_{\xi\phi 1} [(v_2^\phi)^2 - (v_3^\phi)^2] + \frac{1}{2} \lambda_{\xi\phi 2} [(u_2^\phi)^2 - (u_3^\phi)^2] \\
&\quad + \frac{1}{2\sqrt{3}} \lambda_{\xi\phi 12} (2v_1^\phi u_1^\phi - v_2^\phi u_2^\phi - v_3^\phi u_3^\phi) = 0 \\
j) \quad \partial W / \partial f_2^\xi &= \frac{1}{\sqrt{2}} M_{\xi\eta} z_2^\eta + \frac{1}{2} \lambda_{\xi\eta} (z_1^\eta v_1^\varphi - z_2^\eta v_2^\varphi) + \frac{1}{2\sqrt{3}} \lambda_{\xi\phi 1} [-2(v_1^\phi)^2 + (v_2^\phi)^2 + (v_3^\phi)^2] \\
&\quad + \frac{1}{2\sqrt{3}} \lambda_{\xi\phi 2} [-2(u_1^\phi)^2 + (u_2^\phi)^2 + (u_3^\phi)^2] + \frac{1}{2} \lambda_{\xi\phi 12} (v_2^\phi u_2^\phi - v_3^\phi u_3^\phi) = 0 \\
k) \quad \partial W / \partial f_1^{\phi 1} &= \frac{1}{\sqrt{3}} (\lambda_{\xi\phi 12} u_1^\phi u_1^\xi - 2\lambda_{\xi\phi 1} u_2^\xi v_1^\phi) = 0 \\
l) \quad \partial W / \partial f_2^{\phi 1} &= u_1^\xi (\lambda_{\xi\phi 1} v_2^\phi - \frac{1}{2\sqrt{3}} \lambda_{\xi\phi 12} u_2^\phi) + u_2^\xi (\frac{\lambda_{\xi\phi 1}}{\sqrt{3}} v_2^\phi + \frac{1}{2} \lambda_{\xi\phi 12} u_2^\phi) = 0 \\
m) \quad \partial W / \partial f_3^{\phi 1} &= u_1^\xi (-\lambda_{\xi\phi 1} v_3^\phi - \frac{1}{2\sqrt{3}} \lambda_{\xi\phi 12} u_3^\phi) + u_2^\xi (\frac{\lambda_{\xi\phi 1}}{\sqrt{3}} v_2^\phi - \frac{1}{2} \lambda_{\xi\phi 12} u_2^\phi) = 0 \\
n) \quad \partial W / \partial f_1^{\phi 2} &= \frac{1}{\sqrt{3}} (\lambda_{\xi\phi 12} v_1^\phi u_1^\xi - 2\lambda_{\xi\phi 2} u_2^\xi u_1^\phi) = 0 \\
o) \quad \partial W / \partial f_2^{\phi 2} &= u_1^\xi (\lambda_{\xi\phi 2} u_2^\phi - \frac{1}{2\sqrt{3}} \lambda_{\xi\phi 12} v_2^\phi) + u_2^\xi (\frac{\lambda_{\xi\phi 1}}{\sqrt{3}} u_2^\phi + \frac{1}{2} \lambda_{\xi\phi 12} v_2^\phi) = 0 \\
p) \quad \partial W / \partial f_3^{\phi 2} &= u_1^\xi (-\lambda_{\xi\phi 2} u_3^\phi - \frac{1}{2\sqrt{3}} \lambda_{\xi\phi 12} v_3^\phi) + u_2^\xi (\frac{\lambda_{\xi\phi 2}}{\sqrt{3}} u_2^\phi - \frac{1}{2} \lambda_{\xi\phi 12} v_2^\phi) = 0
\end{aligned} \tag{34}$$

Eq. *h*) of eq. (34) implies  $z_{1,2}^\eta = 0$ . As first consequence we have that a possible solution of eqs. *f*) – *g*) and eqs. *k*) – *p*) is given by

$$(u_1^\xi, u_2^\xi) = (0, 0) \quad \text{and} \quad v_\sigma \neq 0. \tag{35}$$

By substituting  $(z_1^\eta, z_2^\eta) = (0, 0)$ ,  $(u_1^\xi, u_2^\xi) = (0, 0)$  and  $v_\sigma \neq 0$  in the equations not yet solved it is easy to check that a possible solution for eqs. *a*) – *e*) is given by the vacuum configuration

$$\begin{aligned}
(v_1^\varphi, v_2^\varphi) &= (0, v^\varphi) \quad \text{with} \quad v^\varphi = \frac{M_\Delta}{\lambda_\Delta} \\
(v_1^\Delta, v_2^\Delta, v_3^\Delta) &= (v^\Delta, 0, 0) \quad \text{with} \quad v^\Delta = 6^{1/4} \frac{\sqrt{M_\varphi M_\Delta}}{\lambda_\Delta}.
\end{aligned} \tag{36}$$

Finally eqs. *i*) – *j*) are solved by the vacuum configuration

$$(v_1^\phi, v_2^\phi, v_3^\phi) = v^\phi (1, 1, 1) \quad \text{and} \quad (u_1^\phi, u_2^\phi, u_3^\phi) = u^\phi (1, 1, 1). \tag{37}$$



The solution found is not unique but can be stabilized once we add apposite SUSY soft breaking terms. In sec. III A we have assumed that the flavon vevs is of order  $\lambda^2 \Lambda$ . Therefore the next to leading order corrections to the Yukawa superpotential induced by the driving fields are sufficiently suppressed.

### B. Model II : minimization of the potential

	$\hat{\Delta}$	$\hat{\sigma}$	$\hat{\phi}$	$\hat{\varphi}$	$\hat{\bar{\sigma}}$	$\hat{\xi}$	$\hat{\eta}$
$SU(2)$	1	1	1	1	1	1	1
$S_4$	3 <sub>1</sub>	1	2	2	1	1	1
$Z_3$	$\omega$	1	1	1	1	$\omega$	$\omega^2$
$Z_5$	1	$\omega_5^2$	$\omega_5^2$	1	$\omega_5$	1	1

TABLE IV: Scalar content of model I including both flavon and the driving field supermultiplets.

The flavon potential is obtained by the following part of the full superpotential

$$\begin{aligned} \mathcal{W} = & \lambda_{\Delta\xi} \hat{\xi} \hat{\Delta} \hat{\Delta} + \lambda_{\Delta} \hat{\Delta} \hat{\Delta} \hat{\Delta} + M_{\xi} \hat{\xi} \hat{\eta} + \lambda_{\xi} \hat{\xi} \hat{\xi} \hat{\xi} + \lambda_{\eta} \hat{\eta} \hat{\eta} \hat{\eta} \\ & + M_{\varphi} \hat{\varphi} \hat{\varphi} + \lambda_{\varphi} \hat{\varphi} \hat{\varphi} \hat{\varphi} + \lambda_{\phi} \hat{\sigma} \hat{\phi} \hat{\phi} + \lambda_{\sigma} \hat{\sigma} \hat{\sigma} \hat{\sigma}. \end{aligned} \quad (38)$$

By assuming the general vacuum configuration

$$\langle \Delta \rangle = (v_1^{\Delta}, v_2^{\Delta}, v_3^{\Delta}), \langle \varphi \rangle = (v_1^{\varphi}, v_2^{\varphi}), \langle \phi \rangle = (v_1^{\phi}, v_2^{\phi}), \langle \xi \rangle = v_{\xi}, \langle \eta \rangle = v_{\eta}, \langle \sigma \rangle = v_{\sigma}, \langle \bar{\sigma} \rangle = v_{\bar{\sigma}}, \quad (39)$$

the minimization of the scalar potential obtained in the SUSY limit gives the following set of equations

$$\begin{aligned} \partial \mathcal{W}_Y / \partial f_1^{\Delta} &= \sqrt{2} \lambda_{\Delta\xi} v_{\xi} v_1^{\Delta} + \sqrt{3} \lambda_{\Delta} v_2^{\Delta} v_3^{\Delta} = 0 \\ \partial \mathcal{W}_Y / \partial f_2^{\Delta} &= \sqrt{2} \lambda_{\Delta\xi} v_{\xi} v_2^{\Delta} + \sqrt{3} \lambda_{\Delta} v_1^{\Delta} v_3^{\Delta} = 0 \\ \partial \mathcal{W}_Y / \partial f_3^{\Delta} &= \sqrt{2} \lambda_{\Delta\xi} v_{\xi} v_3^{\Delta} + \sqrt{3} \lambda_{\Delta} v_1^{\Delta} v_2^{\Delta} = 0 \\ \partial \mathcal{W}_Y / \partial f^{\xi} &= \sqrt{3} \lambda_{\Delta\xi} [(v_1^{\Delta})^2 + (v_2^{\Delta})^2 + (v_3^{\Delta})^2] + M_{\xi} v_{\eta} + 3 \lambda_{\xi} v_{\xi}^2 = 0 \\ \partial \mathcal{W}_Y / \partial f^{\eta} &= M_{\xi} v_{\xi} + 3 \lambda_{\eta} v_{\eta}^2 = 0 \\ \partial \mathcal{W}_Y / \partial f_1^{\varphi} &= \sqrt{2} M_{\varphi} v_1^{\varphi} + 3 \lambda_{\varphi} v_1^{\varphi} v_2^{\varphi} = 0 \\ \partial \mathcal{W}_Y / \partial f_2^{\varphi} &= \sqrt{2} M_{\varphi} v_2^{\varphi} + \frac{3}{2} \lambda_{\varphi} [(v_1^{\varphi})^2 - (v_2^{\varphi})^2] = 0 \\ \partial \mathcal{W}_Y / \partial f_1^{\phi} &= \sqrt{2} \lambda_{\phi} v_1^{\phi} v_{\bar{\sigma}} = 0 \\ \partial \mathcal{W}_Y / \partial f_2^{\phi} &= \sqrt{2} \lambda_{\phi} v_2^{\phi} v_{\bar{\sigma}} = 0 \\ \partial \mathcal{W}_Y / \partial f^{\sigma} &= 2 \lambda_{\bar{\sigma}} v_{\sigma} v_{\bar{\sigma}} = 0 \\ \partial \mathcal{W}_Y / \partial f^{\bar{\sigma}} &= \frac{1}{\sqrt{2}} \lambda_{\phi} [(v_1^{\phi})^2 + (v_2^{\phi})^2] + \lambda_{\bar{\sigma}} v_{\sigma}^2 = 0. \end{aligned} \quad (40)$$

Discarding for the triplet and the doublets the trivial solutions that do not break  $S_4$ , the solution of the system of eq. (40) is given by the following vacuum configuration

$$\begin{aligned} v_1^{\Delta} = v_2^{\Delta} = v_3^{\Delta} = v^{\Delta} \quad & \text{with} \quad v^{\Delta} = \sqrt{2} \frac{\lambda_{\Delta\xi} \lambda_{\eta}}{\lambda_{\Delta}} \frac{v_{\eta}^2}{M_{\xi}} \\ v_{\xi} = -3 \lambda_{\eta} \frac{v_{\eta}^2}{M_{\xi}} \quad & \text{with} \quad v_{\eta}^3 = -M_{\xi}^3 \frac{\lambda_{\Delta}^2}{\lambda_{\eta}^2 (2\sqrt{3} \lambda_{\Delta\xi}^3 + 27 \lambda_{\xi} \lambda_{\Delta}^2)} \\ (v_1^{\varphi}, v_2^{\varphi}) \neq (0, 0) \quad & \text{with} \quad \begin{cases} (0, \frac{2\sqrt{2}}{3} \frac{M_{\varphi}}{\lambda_{\varphi}}) \\ (\sqrt{\frac{2}{3}} \frac{M_{\varphi}}{\lambda_{\varphi}}, -\frac{\sqrt{2}}{3} \frac{M_{\varphi}}{\lambda_{\varphi}}) \\ (-\sqrt{\frac{2}{3}} \frac{M_{\varphi}}{\lambda_{\varphi}}, -\frac{\sqrt{2}}{3} \frac{M_{\varphi}}{\lambda_{\varphi}}) \end{cases} \\ v_{\sigma}^2 = -\frac{1}{\sqrt{2}} \frac{\lambda_{\phi}}{\lambda_{\sigma}} [(v_1^{\phi})^2 + (v_2^{\phi})^2] \neq 0 \text{ and } v_{\bar{\sigma}} = 0. \end{aligned} \quad (41)$$

The three solutions corresponding to  $\langle\varphi\rangle$  are degenerate and corresponding to the breaking of  $S_3$  to its 3 different  $Z_2$  subgroups. Through appropriate choices of soft terms that break the discrete abelian symmetry  $Z_3$  and  $Z_5$  and not  $S_4$  we can stabilize as absolute minimum the vacuum configuration  $\langle\varphi\rangle \sim (0, 1)$ .

## V. CONCLUSION

In this paper we have discussed the idea that  $S_4$  is the minimal discrete non abelian group naturally related to TBM in the lepton sector. We have shown that  $S_4$  can yield exact TBM according to a general group theory analysis and we have presented two explicit model realizations of how TBM can be obtained in  $S_4$  once the basis of its generators are fixed. In addition we have provided a detailed study of the corresponding scalar potentials. The two models require two triplets with different vev alignments. For each model we have built a potential that in the SUSY limit contains the minimum required. The problem of the triplet and doublet alignments is solved in a more economical way than in models based on  $A_4$  [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. To separate the charged lepton sector from the neutrino one we have introduced extra abelian symmetries. The construction of the potentials have not required additional symmetries than such extra abelian symmetries, but just the addition of “driving” fields that do not enter in the Yukawa part. We have studied neither the quark sector nor the possibility to embed such a model in a GUT theory. We leave these subjects for a future publication. It is worth to mention that in  $S_4$  there is more freedom to generate the mixing in the quark sector than in  $A_4$ . Indeed the doublet irreducible representation could play an important role as happens in  $T'$  [30].

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